PRE-BOARD / XII /MATHEMATICS / 2020-21

Time: 3 Hrs. **M M: 80**

General Instructions:

1. This question paper contains two **parts A and B**. Each part is compulsory. Part A carries **24** marks and Part B carries **56** marks
2. **Part-A** has Objective Type Questions and **Part -B** has Descriptive Type Questions
3. Both Part A and Part B have choices.

Part – A:

1. It consists of two sections**- I and II**.
2. Section **I** comprises of 16 very short answer type questions.
3. Section **II** contains **2** case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt **any 4 out of 5 MCQs**.

Part – B:

1. It consists of three sections**- III, IV and V.**
2. Section **III** comprises of 10 questions of **2 marks** each**.**
3. Section **IV** comprises of 7 questions of **3 marks** each**.**
4. Section **V** comprises of 3 questions of **5 marks** each**.**
5. Internal choice is provided in **3** questions of Section –III, **2** questions of Section-IV and **3** questions of Section-V. You have to attempt only one of the alternatives in all such questions.
6. Section-V. You have to attempt only one of the alternatives in all such questions.
7. Questions involving graph paper should be done on the answer sheet only.

PART – A

SECTION I

All questions are compulsory. In case of internal choices attempt anyone.

Q1. A relation R_1 in R is defined as R_1 = {(x, y): $x^2+y^2=9, x, y \in R$}. Check whether R_1 is symmetric or not. 1

Q2. If $f: R \to R$ is a function defined as $f(x)=x^3+1$. Check whether f is onto or not. 1

Q3. A function $f: R-\{0\} \to R$ is defined as $f(x)=x^{\frac{1}{3}}+\frac{1}{x^{\frac{1}{3}}}$. Check whether the function f is injective or not. 1

Q4. Find the number of reflective relations on a set $A=\{1, a, b, c\}$. 1

OR

A relation R in R is defined as R = {(x, y): $2x-2y+\sqrt{11}$ *is irrational*, $x, y \in R$}. Check whether R is transitive or not.

Q5. Find the value of $\sin^{-1}(\sin(5))$. 1

Q6. Using determinants, find the equation of a straight line passing through the points $\left(2, \frac{1}{2}\right)$ & $\left(3, \frac{1}{3}\right)$. 1

Q7. Evaluate: $\int e^x\left(\log(x)+\frac{1}{x^2}\right)dx$ 1

Q8. Find the area of the region bounded by the curve y = $\sqrt{x-3}$, the lines x = 4, x = 7 and x-axis. 1

OR

Find the area of the region curve y = e^x, the lines $x=\log_e(2)$, $x=\log_e(4)$ and x-axis.

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Q9. For what value of n is the following a homogeneous differential equation: 1

$y\frac{dy}{dx} = x + \sqrt[3]{x^3 + y^{n-4}}$.

Q10. Find a unit vector parallel to the sum of the vectors $\hat{i} - \hat{j} - 2\hat{k}$ & $\hat{i} - \hat{j} + \hat{k}$. 1

Q11. Find the projection of $5\hat{j} - 3\hat{k}$ on $\hat{i} + \hat{j} + \hat{k}$. 1

OR

Find the area of a parallelogram whose diagonal vectors are represented by $3\hat{i} - \hat{j} + 4\hat{k}$ and $2\hat{i} - 4\hat{k}$.

Q12. If $|\vec{a}| = 2$ and $|\vec{b}| = 3$, then find the value of $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2$. 1

Q13. Find the equation of a straight line passing through the points (1, 2, 5) and (2, -1, 4) in Cartesian form and vector form. 1

Q14. If a line makes an angle α with x-axis, β with y-axis and γ with z-axis then find the value of $\cos(2\alpha) + \cos(2\beta) + \cos(2\gamma)$. 1

Q15. A card is drawn from a pack of 52 cards. What is the probability that the card is a club, given the card is higher than a 10? 1

Q16. A speaks truth in 70% of the cases and B speaks truth in 60% of the cases. In what percentage they contradict each other in narrating the same statement? 1

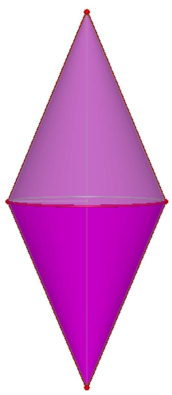
OR

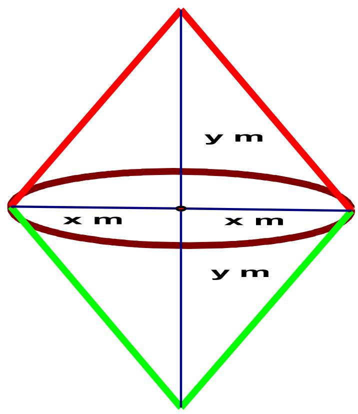
The probability of student A passing an examination is 2/7 and of student B passing is 4/7.Find the probability of at least one of them passing the examination.

SECTION II

Both the Case study-based questions are compulsory. Attempt any 4 sub parts from each question (17-18)

Q17. A can buoy in the form of a double cone is to be made from two equal circular iron plates such that slant height of buoy is 3 m. (use π = 3.14 m)





Assume x m is the radius of the base and 2y m is the whole height of the buoy.

(i) The relation between x and y is: 1

a) $y^2 = 9 + x^2$ b) $x^2 = 9 + y^2$ c) $y = \sqrt{9 - x^2}$ d) None of these

(ii) Volume of the buoy in terms of the variable y is: 1

a) $\frac{2\pi}{3}(9y - y^2)\ m^3$ b) $\frac{\pi}{3}(9y - y^3)\ m^3$ c) $\frac{2\pi}{3}(9y + y^3)\ m^3$ d) $\frac{2\pi}{3}(9y - y^3)\ m^3$

(iii) Curved surface area of the buoy in terms of the variable y is: 1

a) $6\pi\sqrt{9-y^2}\ m^2$ b) $6\pi\sqrt{y^2-9}\ m^2$ c) $6\pi\sqrt{y^2+9}\ m^2$ d) None of these

(iv) Height of the buoy when the volume is maximum is: 1

a) $\sqrt{3}\ m$ b) $2\sqrt{3}\ m$ c) $4\sqrt{3}\ m$ d) None of these

(v) Cost to paint the buoy @ ₹ 50 per square metre when the volume is maximum is: 1

a) ₹ 2308 b) ₹ 2309 c) ₹ 2310 d) None of these

Q18. An insurance company divides its policy holders into three categories: low risk, moderate risk, and high risk**.** The low-risk policy holders account for 60% of the total number of people insured by the company. The moderate-risk policy holders account for 30%, and the high-risk policy holders account for 10%. The probabilities that a low-risk, moderate-risk, and high-risk policy holder will file a claim within a given year are respectively .01, .10, and .50.

(i) The conditional probability that policy holder will file a claim within a given year given that policy holder is a high-risk policy holder: 1

a) 0.05 b) 0.1 c) 0.5 d) None of these

(ii) Probability that low-risk policy holder will file a claim within a given year is: 1

a) .01 b) .06 c) 0.0006 d) None of these

(iii) The total probability of policy holder will file a claim within a given year is: 1

a) 0.086 b) 0.087 c) 0.088 d) None of these

(iv) Given that a policy holder files a claim this year, the probability that the person is a high-risk policy holder is: 1

a) 50/43 b) 25/43 c) 15/43 d) 3/43

(v) Let E_1 = Event that person is low-risk policy holder. 1

E_2 = Event that person is moderate risk policy holder.

E_3 = Event that person is high risk policy holder.

A = Event that a policy holder will file a claim this year.

The value of $\sum_{k=1}^{3} P\left(E_k / A\right)$ is

a) 1 b) 1/2 c) 1/3 d) None of these

PART – B
SECTION III
(19-28). Each question carries 2 mark. In case of internal choices attempt anyone.

Q19. Find all values of a, b, c, for which the matrix A is symmetric: 2

$$A = \begin{bmatrix} 2 & a-2b+2c & 2a+b+c \\ 3 & 5 & a+c \\ 0 & -2 & 7 \end{bmatrix}.$$

OR

Construct a matrix of order 2×2 where $a_{ij} = \dfrac{(i-2j)^2}{2}$. 2

Q20. Find the matrix 'X' such that $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \cdot X + \begin{bmatrix} -5 & 0 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 3 & -9 \\ 7 & 1 \end{bmatrix}$. 2

Q21. The function $f(x) = \dfrac{\log(1+\alpha x) - \log(1-\beta x)}{x}$ is not defined at x = 0. Find the value of $f(0)$ so that $f(x)$ may be continuous. 2

Q22. Find the equation of the tangent to the curve $y = \dfrac{6x}{x^2-1}$ at the point (2,4). 2

Q23. Evaluate: $\displaystyle\int \frac{\sqrt{\cot x}}{\sin x \cos x}\,dx$ 2

OR

Evaluate: $\displaystyle\int_2^5 \log\left(\frac{x}{7-x}\right)dx.$ 2

Q24. Find the area of the region bounded by the curve $y^2 - 6y + x + 5 = 0$ and y-axis. 2

Q25. Solve: $\sqrt{1-x^3}\,dy = \sqrt{x}\,dx,\ y(1) = \pi$. 2

Q26. If $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $|\vec{a}| = 2, |\vec{b}| = 3$ and $|\vec{c}| = 6$, each of these vectors is perpendicular to the sum of other two vectors then find $|\vec{a} + \vec{b} + \vec{c}|$. 2

Q27. Using vector method, prove that the parallelograms lying on the same base and between the same parallels are equal in area. 2

Q28. Suppose we have two urns. Urn I contains 12 green balls, 16 yellow, and 2 red. Urn II contains 6 green balls, 5 yellow, and 4 red. We flip a coin. If a head we draw one ball randomly from Urn I, if a tail we draw one ball randomly from Urn II. Find the probability that yellow ball is drawn. 2

OR

Two friends X and Y have equal number of sons. There are three tickets for a football match, which are to be distributed among sons. The probability that 2 tickets go to the sons of one and one ticket goes to the son of other is 6/7. Find how many sons each of the two friends have. 2

SECTION IV
All questions are compulsory. In case of internal choices attempt anyone

Q29. A relation R in set of all-natural numbers N is defined as aRb if and only if a + b is even. Prove that R is an equivalence Relation. Also write the equivalence class of 1. 3

Q30. If $y = x^{\sin(x)} + \left(\sin(x)\right)^x$, then find $\frac{dy}{dx}$. 3

OR

Show that $f(x) = |x-1| + |x-5|$ is not differentiable at x = 1. 3

Q31. If $y = \left(x + \sqrt{x^2+1}\right)^m$, prove that $\left(1+x^2\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2 y = 0$. 3

Q32. Determine the intervals in which the function $f(x) = x^{\frac{2}{3}}\left(\frac{5}{2} - x\right)$ is increasing or decreasing. 3

Q33. Evaluate: $\int \frac{x^2+9}{\left(x^2+1\right)\left(x^2+4\right)}dx$.. 3

Q34. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. 3

Q35. Solve: $\sin(x)\frac{dy}{dx} + \sin(2x) = \sin(3x)$ 3

OR

Solve: $\left(x^3 + y^3\right)dy - x^2 y dx = 0$.

SECTION V

All questions are compulsory. In case of internal choices attempt any one

Q36. If $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{pmatrix}$ then find AB and hence solve: 5

$-3x + 2y + 2z = 7; 2x - 3y + 2z = 2; 2x + 2y - 3z = -3$.

OR

If $A = \begin{bmatrix} 1 & 3 & 3 \\ 3 & 1 & 3 \\ 3 & 3 & 1 \end{bmatrix}$ then prove that $A^2 - 5A - 14I = O$ and hence find A^{-1}.

Q37. Show that the lines $\frac{x-1}{2} = \frac{y+3}{3} = \frac{z-1}{1} = m$ and $\frac{x+1}{2} = \frac{y+14}{5} = \frac{z-8}{-1} = n$ intersect. Find the coordinates of their point of intersection. 5

OR

Find the distance of the point (3, -4, 5) from the plane 2x + 5y – 6z = 16, measured parallel to the line $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z+3}{-2}$.

Q38. Solve the following linear programming problem (L.P.P) graphically. 5

Minimise $Z = 5x + 10y$ subject to the constraints:

$x + 2y \le 120$

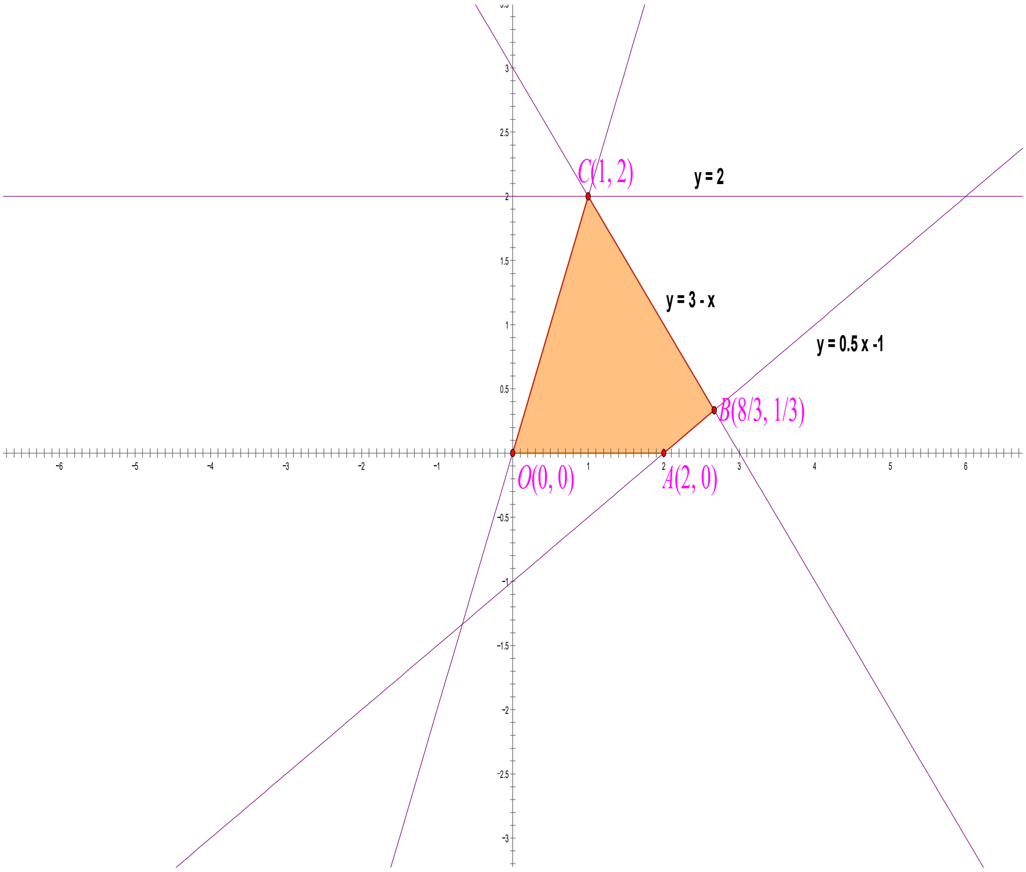
$x + y \ge 60$

$x - 2y \le 0$

$x \ge 0,\ y \ge 0$

OR

The corner points of the feasible region determined by the system of linear constraints are as shown below:



Answer each of the following:

(i) Let $Z = 3x - 4y$ be the objective function. Find the maximum and minimum value of Z and also the corresponding points at which the maximum and minimum value occurs.

(ii) Let □=□□ + □□, where □, □ > 0 be the objective function. Find the condition on □ and □ so that the maximum value of Z occurs at B (8/3,1/3) *and* C (1,2). Also mention the number of optimal solutions in this case.